For a Half Wave Rectifier Output

$$I_{av} = \frac{I_m}{\pi} = 0.318 I_m$$

$$I_{rms} = \frac{I_m}{2} = 0.5 I_m$$

$$FF = \frac{I_{rms}}{I_{av}} = \frac{0.5 I_m}{0.318 I_m} = 1.57$$

$$PF = \frac{I_m}{I_{rms}} = \frac{I_m}{0.5 I_m} = 2$$

Phasor Representation

An alternating quantity can be represented using

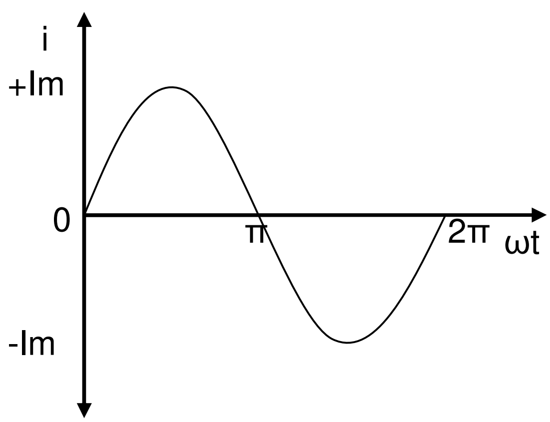
(i) Waveform

(ii) Equations

(iii) Phasor

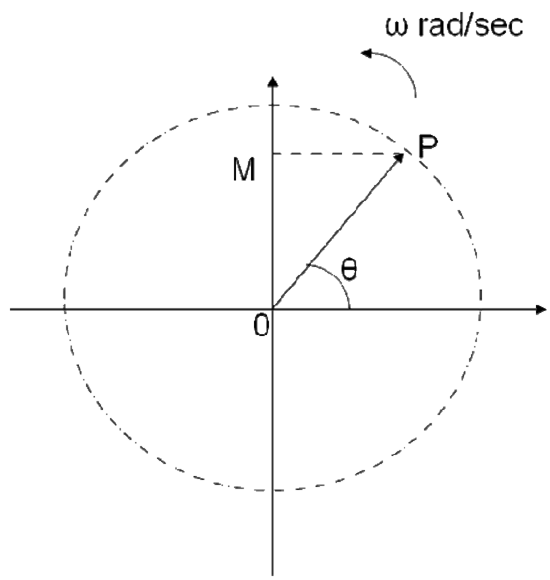
A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.** A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.

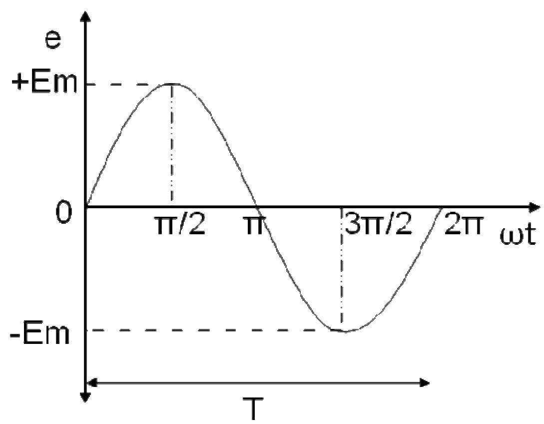


$$i = I_m \sin \omega t$$

Draw a line OP of length equal to I_m. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of OP on the y-axis is given by $OM = OP\sin\theta = I_m \sin\omega t$. Hence the line OP is the phasor representation of the sinusoidal current



Phase

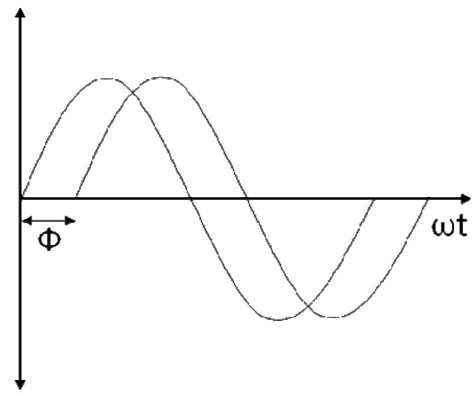


Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference

Phase of $+E_m$ is $\pi/2$ rad or T/4 sec

Phase of $-E_m$ is $3\pi/2$ rad or 3T/4 sec

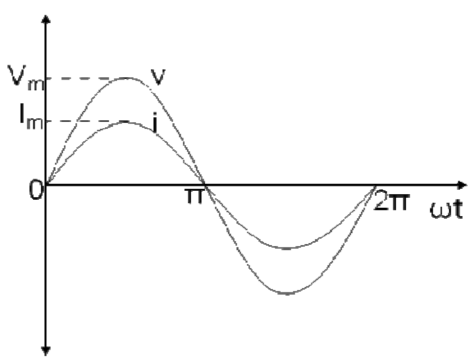
Phase Difference

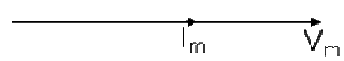


When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.



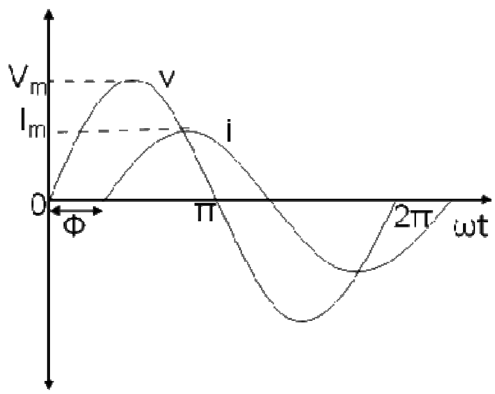


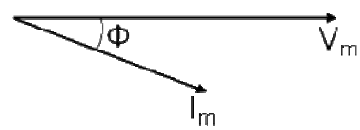
$$v = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

Lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.



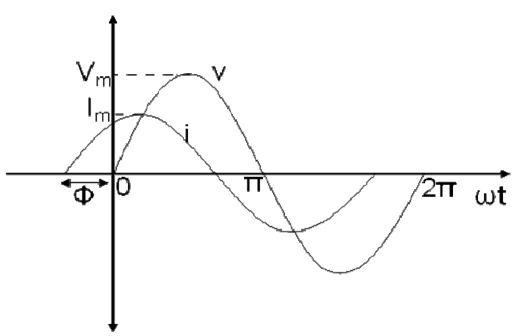


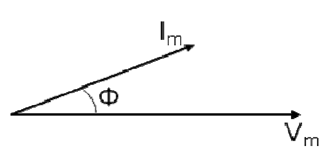
$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \Phi)$$

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.

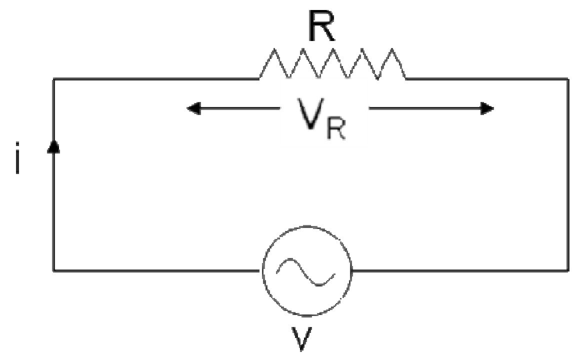




$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \Phi)$$

AC circuit with a pure resistance



Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the resistor is given as V_R which is the same as v.

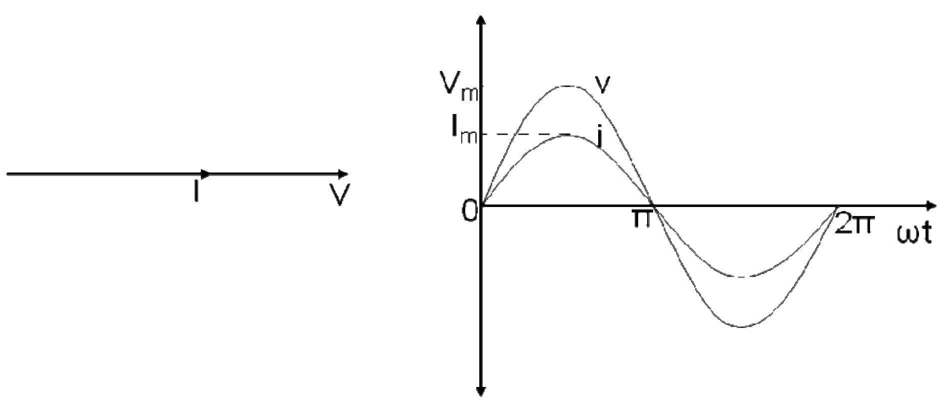
Using ohms law, we can write the following relations

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

$$i = I_m \sin \omega t \qquad \text{---------------(2)}$$

Where $I_m = \dfrac{V_m}{R}$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasors can be drawn as below.



Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin \omega t)$$

$$p = V_m I_m \sin^2 \omega t$$

$$p = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$

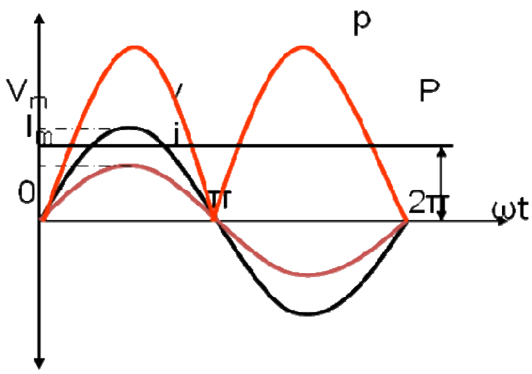
$$P = \frac{V_m I_m}{2} - \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$$

$$P = V.I$$

As seen above the average power is the product of the rms voltage and the rms current.

The voltage, current and power waveforms of a purely resistive circuit is as shown in the figure.



As seen from the waveform, the instantaneous power is always positive meaning that the power always flows from the source to the load.

Phasor Algebra for a pure resistive circuit

$$\overline{V} = V\angle 0^\circ = V + j0$$

$$\overline{I} = \frac{\overline{V}}{R} = \frac{V + j0}{R} = I + j0 = I\angle 0^\circ$$

Problem 2

An ac circuit consists of a pure resistance of 10Ω and is connected to an ac supply of 230 V, 50 Hz. Calculate the (i) current (ii) power consumed and (iii) equations for voltage and current.

$$(i) I = \frac{V}{R} = \frac{230}{10} = 23A$$

$$(ii) P = VI = 230 \times 23 = 5260W$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

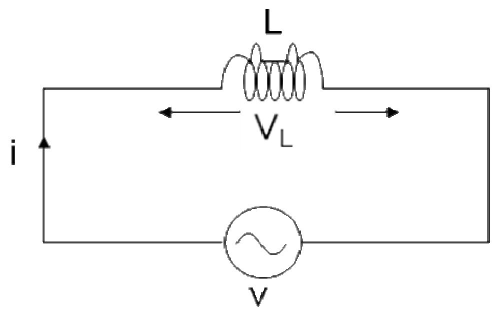
$$I_m = \sqrt{2}I = 32.52A$$

$$\omega = 2\pi f = 314 rad/\sec$$

$$v = 325.25 \sin 314t$$

$$i = 32.52 \sin 314t$$

AC circuit with a pure inductance



Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the inductor is given as V_L which is the same as v.

We can find the current through the inductor as follows

$$v = L\frac{di}{dt}$$

$$V_m \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin \omega t dt$$

$$i = \frac{V_m}{L}\int \sin \omega t dt$$

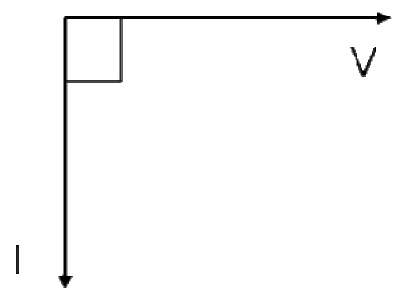
$$i = \frac{V_m}{\omega L}(-\cos \omega t)$$

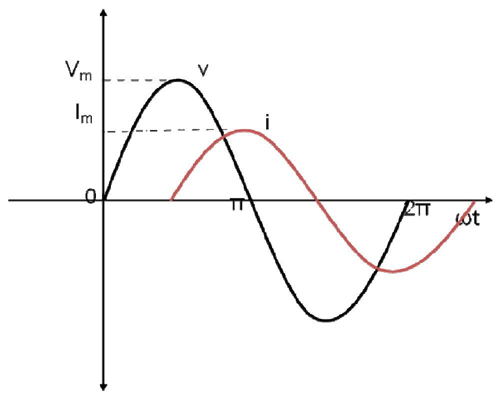
$$i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$$

$$i = I_m \sin(\omega t - \pi/2) \qquad \text{-------------------(2)}$$

Where $I_m = \dfrac{V_m}{\omega L}$

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90^o. Hence the voltage and current waveforms and phasors can be drawn as below.





Inductive reactance

The inductive reactance X_L is given as

$$X_L = \omega L = 2\pi f L$$

$$I_m = \frac{V_m}{X_L}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω)

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \pi / 2))$$

$$p = -V_m I_m \sin \omega t \cos \omega t$$

$$p = -\frac{V_m I_m}{2} \sin 2\omega t$$

As seen from the above equation, the instantaneous power is fluctuating in nature.

Average power

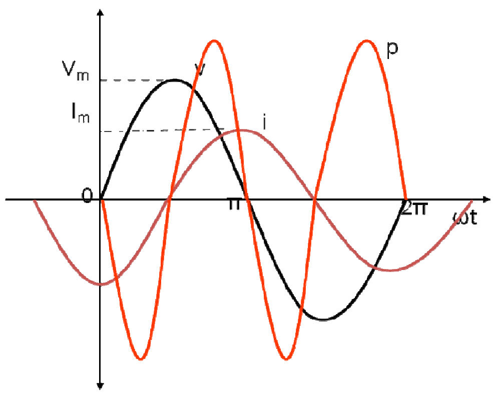
From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2}\sin 2\omega t\, d\omega t$$

$$P = 0$$

The average power in a pure inductive circuit is zero. Or in other words, the power consumed by a pure inductance is zero.

The voltage, current and power waveforms of a purely inductive circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the inductor and when the power in negative, the power flows from the inductor to the source. The positive power is equal to the negative power and hence the average power in the circuit is equal to zero. The power just flows between the source and the inductor, but the inductor does not consume any power.

Phasor algebra for a pure inductive circuit

$$\overline{V} = V\angle 0^\circ = V + j0$$

$$\overline{I} = I\angle -90^\circ = 0 - jI$$

$$\frac{\overline{V}}{\overline{I}} = \frac{V\angle 0^\circ}{I\angle -90} = X_L\angle 90^\circ$$

$$\overline{V} = \overline{I}(jX_L)$$

Problem 3

A pure inductive coil allows a current of 10A to flow from a 230V, 50 Hz supply. Find (i) inductance of the coil (ii) power absorbed and (iii) equations for voltage and current.

$$(i) X_L = \frac{V}{I} = \frac{230}{10} = 23\Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = 0.073H$$

$$(ii) P = 0$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

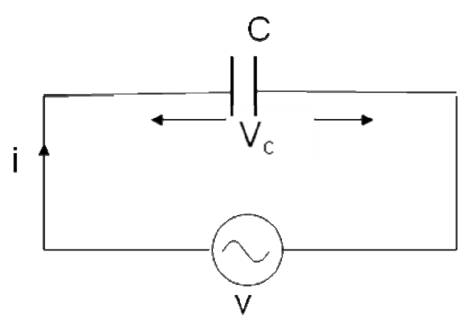
$$I_m = \sqrt{2}I = 14.14A$$

$$\omega = 2\pi f = 314 rad/\sec$$

$$v = 325.25 \sin 314t$$

$$i = 14.14 \sin(314t - \pi/2)$$

AC circuit with a pure capacitance



Consider an AC circuit with a pure capacitance C as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the capacitor is given as V_C which is the same as v.

We can find the current through the capacitor as follows

$$q = Cv$$

$$q = CV_m \sin \omega t$$

$$i = \frac{dq}{dt}$$

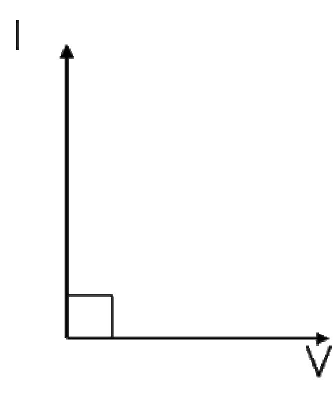
$$i = CV_m \omega \cos \omega t$$

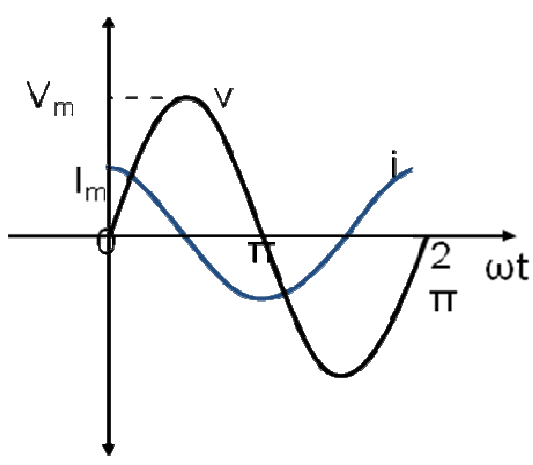
$$i = \omega CV_m \sin(\omega t + \pi / 2)$$

$$i = I_m \sin(\omega t + \pi / 2) \qquad \text{--------------------(2)}$$

Where $I_m = \omega CV_m$

From equation (1) and (2) we observe that in a pure capacitive circuit, the current leads the voltage by 90^{o}. Hence the voltage and current waveforms and phasors can be drawn as below.





Capacitive reactance

The capacitive reactance X_C is given as

$$X_L = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$I_m = \frac{V_m}{X_C}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω)

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t + \pi / 2))$$

$$p = V_m I_m \sin \omega t \cos \omega t$$

$$p = \frac{V_m I_m}{2} \sin 2\omega t$$

As seen from the above equation, the instantaneous power is fluctuating in nature.

Average power

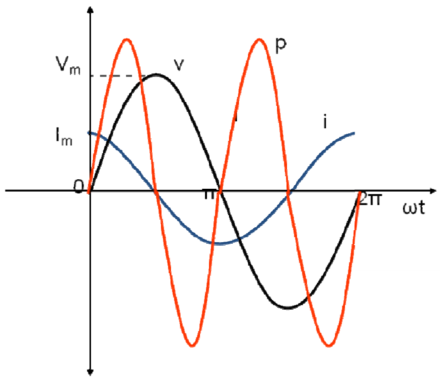
From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t d\omega t$$

$$P = 0$$

The average power in a pure capacitive circuit is zero. Or in other words, the power consumed by a pure capacitance is zero.

The voltage, current and power waveforms of a purely capacitive circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the capacitor and when the power in negative, the power flows from the capacitor to the source. The positive power is equal to the negative power and hence the average power in the circuit is equal to zero. The power just flows between the source and the capacitor, but the capacitor does not consume any power.

Phasor algebra in a pure capacitive circuit

$$\overline{V} = V\angle 0^\circ = V + j0$$

$$\overline{I} = I\angle 90^\circ = 0 + jI$$

$$\frac{\overline{V}}{\overline{I}} = \frac{V\angle 0^\circ}{I\angle 90} = X_C\angle -90^\circ$$

$$\overline{V} = \overline{I}(-jX_C)$$

Problem 4

A 318μF capacitor is connected across a 230V, 50 Hz system. Find (i) the capacitive reactance (ii) rms value of current and (iii) equations for voltage and current.

$$(i) X_C = \frac{1}{2\pi f C} = 10\Omega$$

$$(ii) I = \frac{V}{X_C} = 23A$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

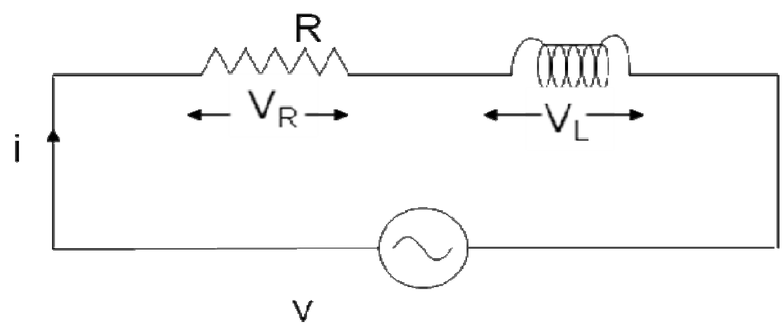
$$I_m = \sqrt{2}I = 32.53A$$

$$\omega = 2\pi f = 314 rad/\sec$$

$$v = 325.25 \sin 314t$$

$$i = 32.53 \sin(314t + \pi/2)$$

R-L Series circuit



Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by

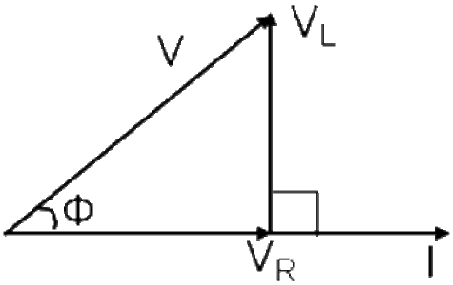
$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the inductor is V_L.

V_R=IR is in phase with I

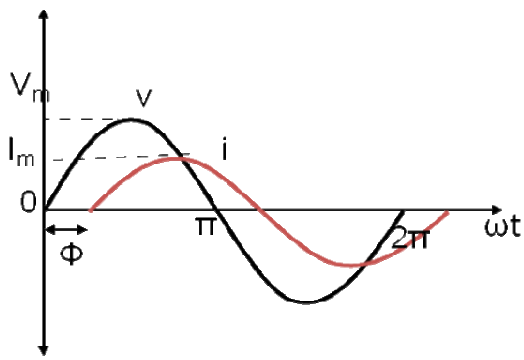
V_L=IX_L leads current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90°. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.

The waveform and equations for an RL series circuit can be drawn as below.



$$V = V_m \sin \omega t$$
$$I = I_m \sin(\omega t - \Phi)$$

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$
$$V_R = IR$$
$$V_L = IX_L$$
$$V = \sqrt{(IR)^2 + (IX_L)^2}$$
$$V = I\sqrt{R^2 + X_L^2}$$
$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + X_L^2}$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms (Ω).

Phase angle

$$\Phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{IX_L}{IR}\right)$$

$$\Phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Instantaneous power

The instantaneous power in an RL series circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \Phi)$$

$$p = \frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

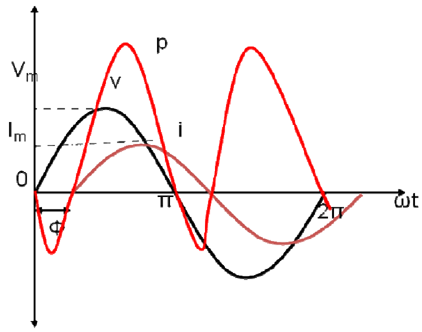
$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)\right]d\omega t$$

$$P = \frac{V_m I_m}{2}\cos\Phi$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\Phi$$

$$P = VI\cos\Phi$$

The voltage, current and power waveforms of a RL series circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the load and when the power in negative, the power flows from the load to the source. The positive power is not equal to the negative power and hence the average power in the circuit is not equal to zero.

From the phasor diagram,

$$\cos\Phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$$P = VI\cos\Phi$$

$$P = (IZ)\times I\times\frac{R}{Z}$$

$$P = I^2R$$

Hence the power in an RL series circuit is consumed only in the resistance. The inductance does not consume any power.

Power Factor

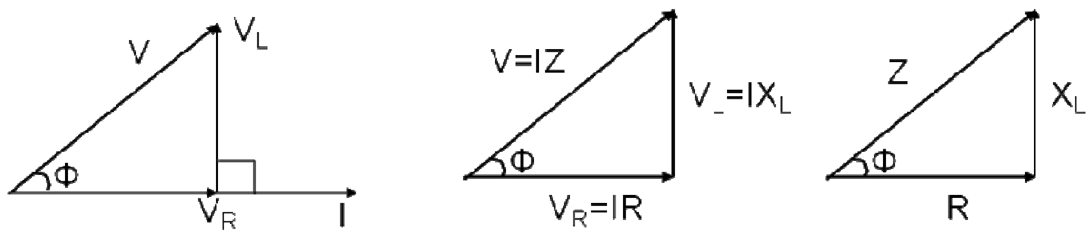
The power factor in an AC circuit is defined as the cosine of the angle between voltage and current ie cosΦ

$$P = VI\cos\Phi$$

The power in an AC circuit is equal to the product of voltage, current and power factor.

Impedance Triangle

We can derive a triangle called the impedance triangle from the phasor diagram of an RL series circuit as shown



The impedance triangle is right angled triangle with R and X_L as two sides and impedance as the hypotenuse. The angle between the base and hypotenuse is Φ. The impedance triangle enables us to calculate the following things.

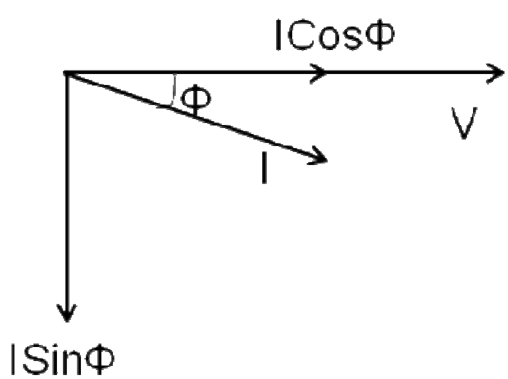
1. Impedance $Z = \sqrt{R^2 + X_L^2}$
2. Power Factor $\cos\Phi = \frac{R}{Z}$
3. Phase angle $\Phi = \tan^{-1}\left(\frac{X_L}{R}\right)$
4. Whether current leads or lags behind the voltage

Power

In an AC circuit, the various powers can be classified as

1. Real or Active power
2. Reactive power
3. Apparent power

Real or active power in an AC circuit is the power that does useful work in the cicuit. Reactive power flows in an AC circuit but does not do any useful work. Apparent power is the total power in an AC circuit.



From the phasor diagram of an RL series circuit, the current can be divided into two components. One component along the voltage IcosΦ, that is called as the active component of current and another component perpendicular to the voltage IsinΦ that is called as the reactive component of current.

Real Power

The power due to the active component of current is called as the active power or real power. It is denoted by P.

$P = V \times I\cos\Phi = I^2R$

Real power is the power that does useful power. It is the power that is consumed by the resistance. The unit for real power in Watt(W).

Reactive Power

The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

$Q = V \times I\sin\Phi = I^2X_L$

Reactive power does not do any useful work. It is the circulating power in th L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

Apparent Power

The apparent power is the total power in the circuit. It is denoted by S.

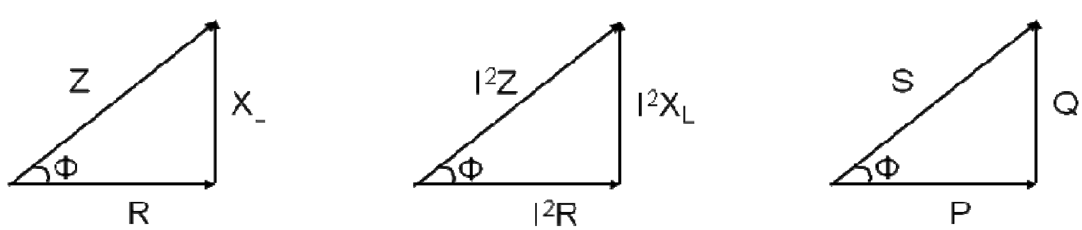
$S = V \times I = I^2Z$

$$S = \sqrt{P^2 + Q^2}$$

The unit for apparent power is Volt Amperes (VA).

Power Triangle

From the impedance triangle, another triangle called the power triangle can be derived as shown.



The power triangle is right angled triangle with P and Q as two sides and S as the hypotenuse. The angle between the base and hypotenuse is Φ. The power triangle enables us to calculate the following things.

1. Apparent power $S = \sqrt{P^2 + Q^2}$
2. Power Factor $Cos\Phi = \frac{P}{S} = \frac{\mathrm{Re}\,alPower}{ApparentPower}$

The power Factor in an AC circuit can be calculated by any one of the following methods

- Cosine of angle between V and I
- Resistance/Impedance R/Z
- Real Power/Apparent Power P/S

Phasor algebra in a RL series circuit

$$V = V + j0 = V\angle 0^\circ$$

$$\overline{Z} = R + jX_L = Z\angle\Phi$$

$$\overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{V}{Z}\angle -\Phi$$

$$\overline{S} = VI^* = P + jQ$$

Problem 5

A coil having a resistance of 7Ω and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed

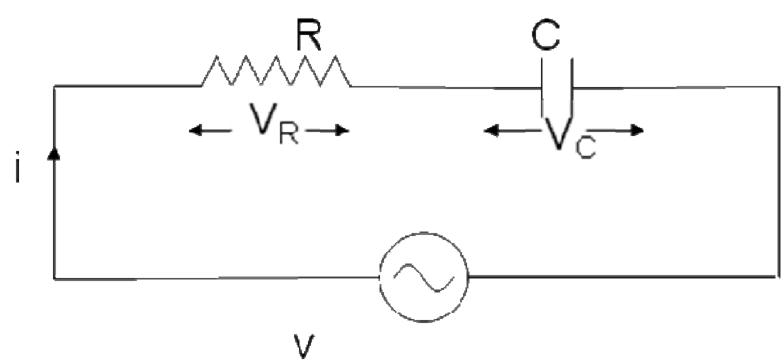
$$X_L = 2\pi fL = 2\times 3.14\times 50\times 31.8\times 10^{-3} = 10\Omega$$
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\Omega$$
$$(i) I = \frac{V}{Z} = \frac{230}{12.2} = 18.85A$$
$$(ii) \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{10}{7}\right) = 55^\circ \, lag$$
$$(iii) PF = \cos\Phi = \cos(55^\circ) = 0.573 lag$$
$$(iv) P = VI\cos\Phi = 230\times 18.85\times 0.573 = 2484.24W$$

Problem 6

A 200 V, 50 Hz, inductive circuit takes a current of 10A, lagging 30 degree. Find (i) the resistance (ii) reactance (iii) inductance of the coil

$$Z = \frac{V}{I} = \frac{200}{10} = 20\Omega$$
$$(i) R = Z\cos\phi = 20\times\cos 30^\circ = 17.32\Omega$$
$$(ii) X_L = Z\sin\phi = 20\times\sin 30^\circ = 10\Omega$$
$$(iii) L = \frac{X_L}{2\pi f} = \frac{10}{2\times 3.14\times 50} = 0.0318H$$

R-C Series circuit



Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

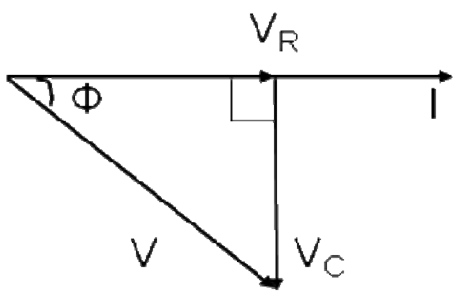
$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the capacitor is V_C.

V_R=IR is in phase with I

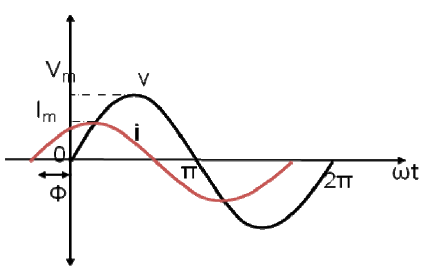
V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags behind the current by 90^o. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

The waveform and equations for an RC series circuit can be drawn as below.



$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t + \Phi)$$

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_C^2}$$
$$V_R = IR$$
$$V_C = IX_C$$
$$V = \sqrt{(IR)^2 + (IX_C)^2}$$
$$V = I\sqrt{R^2 + X_C^2}$$
$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + X_C^2}$

Phase angle

$$\Phi = \tan^{-1}\left(\frac{V_C}{V_R}\right)$$
$$\Phi = \tan^{-1}\left(\frac{IX_C}{IR}\right)$$
$$\Phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$
$$\Phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

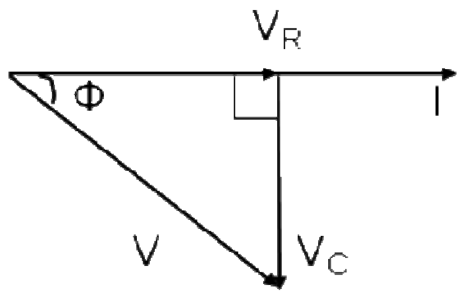
Average power

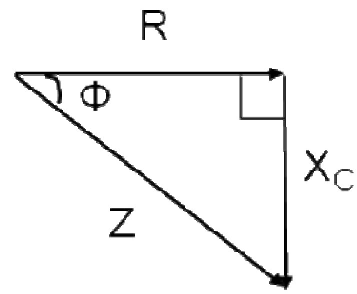
$$P = VI\cos\phi$$
$$P = (IZ)\times I\times\frac{R}{Z}$$
$$P = I^2R$$

Hence the power in an RC series circuit is consumed only in the resistance. The capacitance does not consume any power.

Impedance Triangle

We can derive a triangle called the impedance triangle from the phasor diagram of an RC series circuit as shown





Phasor algebra for RC series circuit

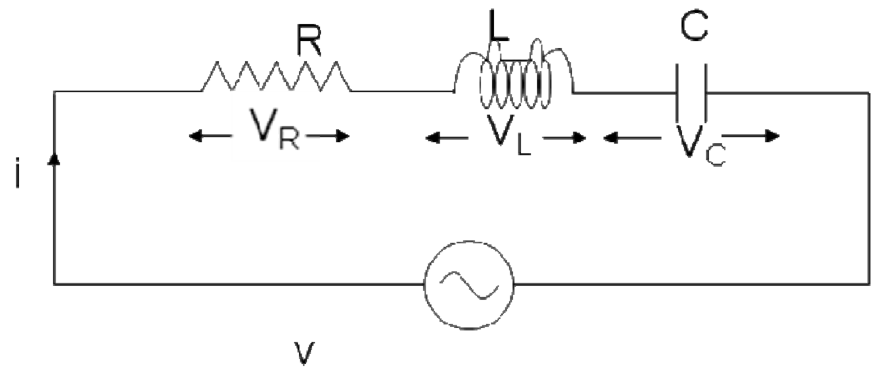
$$V = V + j0 = V\angle 0^\circ$$
$$\overline{Z} = R - jX_C = Z\angle -\Phi$$
$$\bar{I} = \frac{\overline{V}}{\overline{Z}} = \frac{V}{Z}\angle +\Phi$$

Problem 7

A Capacitor of capacitance 79.5μF is connected in series with a non inductive resistance of 30Ω across a 100V, 50Hz supply. Find (i) impedance (ii) current (iii) phase angle (iv) Equation for the instantaneous value of current

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 79.5\times 10^{-6}} = 40\Omega$$
$$(i) Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = 50\Omega$$
$$(ii) I = \frac{V}{Z} = \frac{100}{50} = 2A$$
$$(iii) \Phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{40}{30}\right) = 53^\circ \, lead$$
$$(iv) I_m = \sqrt{2}I = \sqrt{2}\times 2 = 2.828A$$
$$\omega = 2\pi f = 2\times 3.14\times 50 = 314 rad/\sec$$
$$i = 2.828\sin(314t + 53^\circ)$$

R-L-C Series circuit



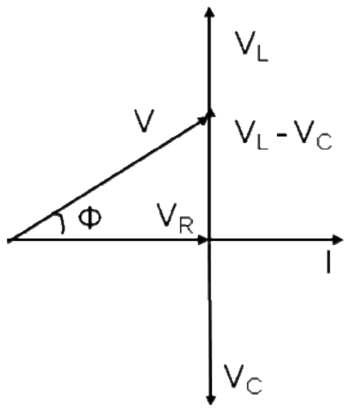
Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t$$

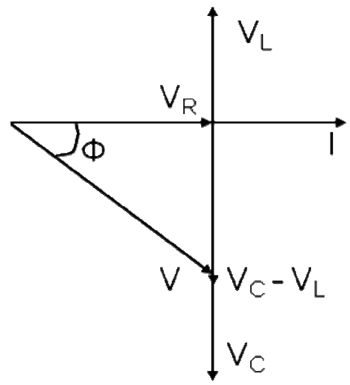
The current flowing in the circuit is i. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C.

V_R=IR is in phase with I
V_L=IX_L leads the current by 90 degrees
V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown. The current I is taken as the reference phasor. The voltage V_R is in phase with I, the voltage V_L leads the current by 90^o and the voltage V_C lags behind the current by 90^o. There are two cases that can occur $V_L>V_C$ and $V_L<V_C$ depending on the values of X_L and X_C. And hence there are two possible phasor diagrams. The phasor V_L-V_C or V_C-V_L is drawn and then the resultant voltage V is drawn.



$V_L > V_C$



$V_L < V_C$

From the phasor diagram we observe that when $V_L > V_C$, the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ. When $V_L < V_C$,the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase angle

$$\Phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{IX_L - IX_C}{IR}\right)$$

$$\Phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

From the expression for phase angle, we can derive the following three cases

Case (i): When $X_L > X_C$

The phase angle Φ is positive and the circuit is inductive. The circuit behaves like a series RL circuit.

Case (ii): When XL<XC

The phase angle Φ is negative and the circuit is capacitive. The circuit behaves like a series RC circuit.

Case (iii): When XL=XC

The phase angle Φ = 0 and the circuit is purely resistive. The circuit behaves like a pure resistive circuit.

The voltage and the current can be represented by the following equations. The angle Φ is positive or negative depending on the circuit elements.

$$V = V_m \sin \omega t$$
$$I = I_m \sin(\omega t \pm \Phi)$$

Average power

$$P = VI \cos\phi$$
$$P = (IZ) \times I \times \frac{R}{Z}$$
$$P = I^2 R$$

Hence the power in an RLC series circuit is consumed only in the resistance. The inductance and the capacitance do not consume any power.

Phasor algebra for RLC series circuit

$$V = V + j0 = V\angle 0^\circ$$
$$\overline{Z} = R + j\left(X_L - X_C\right) = Z\angle\Phi$$
$$\bar{I} = \frac{\overline{V}}{\overline{\overline{Z}}} = \frac{V}{Z}\angle -\Phi$$

Problem 8

A 230 V, 50 Hz ac supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6.8 μF capacitor. Calculate (i) Impedance (ii) Current (iii) Phase angle between current and voltage (iv) power factor (v) power consumed

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 0.06 = 18.84\,\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 6.8\times 10^{-6}} = 468\,\Omega$$

$$(i)Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2.5^2 + (18.84 - 468)^2} = 449.2\Omega$$

$$(ii)I = \frac{V}{Z} = \frac{230}{449.2} = 0.512\,A$$

$$(iii)\Phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{18.84 - 468}{30}\right) = -89.7^\circ$$

$$(iv)\,pf = \cos\Phi = \cos 89.7 = 0.0056\,lead$$

$$(v)P = VI\cos\Phi = 230\times 0.512\times 0.0056 = 0.66W$$

Problem 9

A resistance R, an inductance L=0.01 H and a capacitance C are connected in series. When an alternating voltage v=400sin(3000t-20º)is applied to the series combination, the current flowing is $10\sqrt{2}$ sin(3000t-65º). Find the values of R and C.

$$\Phi = 65^\circ - 20^\circ = 45^\circ\,lag$$

$$X_L = \omega L = 3000\times 0.01 = 30\Omega$$

$$\tan\Phi = \tan 45^\circ = 1$$

$$\tan\Phi = \frac{X_L - X_C}{R} = 1$$

$$R = X_L - X_C$$

$$Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28.3\Omega Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + R^2}$$

$$\sqrt{2}R = 28.3$$

$$R = 20\Omega$$

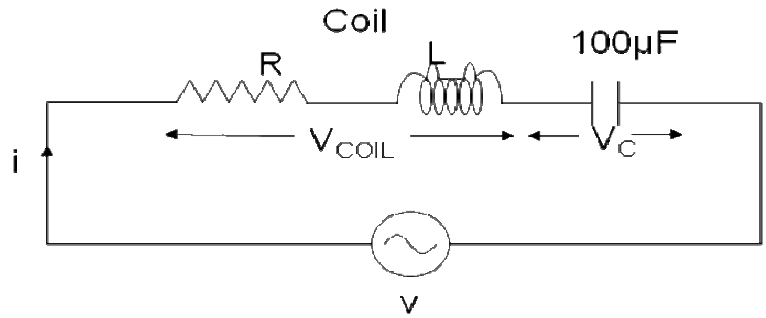
$$X_L - X_C = 20\Omega$$

$$X_C = 30 - 20 = 10\Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{3000\times 10} = 33.3\mu F$$

Problem 10

A coil of pf 0.6 is in series with a 100µF capacitor. When connected to a 50Hz supply, the potential difference across the coil is equal to the potential difference across the capacitor. Find the resistance and inductance of the coil.



$\cos\Phi_{coil} = 0.6$
$C = 100\mu F$
$f = 50Hz$
$V_{coil} = V_c$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times3.14\times50\times100\times10^{-6}} = 31.83\Omega$$

$$V_{coil} = V_c$$

$$IZ_{coil} = IX_C$$

$$Z_{coil} = X_C = 31.83\Omega$$

$$R = Z_{coil}\cos\Phi_{coil} = 31.83\times0.6 = 19.09\Omega$$

$$X_L = \sqrt{Z_{coil}^2 - R^2} = \sqrt{31.83^2 - 19.09^2} = 25.46\Omega$$

$$L = \frac{1}{2\pi fL} = \frac{1}{2\times3.14\times50\times25.46} = 0.081H$$

Problem 11

A current of (120-j50)A flows through a circuit when the applied voltage is (8+j12)V. Determine (i) impedance (ii) power factor (iii) power consumed and reactive power

$$\overline{V} = 8 + j12$$

$$\overline{I} = 120 - j50$$

$$(i)\overline{Z} = \frac{\overline{V}}{\overline{I}} = \frac{8 + j12}{120 - j50} = 0.02 + j0.11 = 0.11\angle 79.7^\circ$$

$$Z = 0.11\Omega$$

$$\Phi = 79.7^\circ$$

$$(ii)\, pf = \cos\Phi = \cos 79.7^\circ = 0.179 lag$$

$$(iii) S = VI^* = (8 + j12)\times(120 + j50) = 360 + j1840$$

$$S = P + jQ$$

$$P = 360W$$

$$Q = 1840VAR$$

Problem 12

The complex Volt Amperes in a series circuit are (4330-j2500) and the current is (25+j43.3)A. Find the applied voltage.

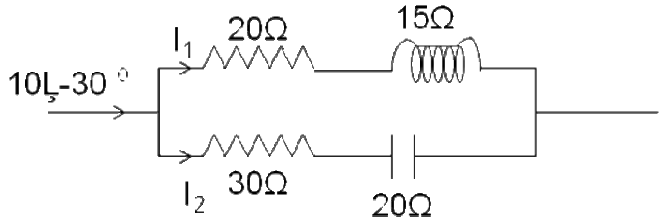
$$\overline{S} = 4330 + j2500$$

$$\overline{I} = 25 + j43.3$$

$$\overline{V} = \frac{\overline{S}}{\overline{I^*}} = \frac{4330 + j2500}{25 - j43.3} = 86.6 + j50$$

Problem 13

A parallel circuit comprises of a resistor of 20Ω in series with an inductive reactance 15Ω in one branch and a resistor of 30Ω in series with a capacitive reactance of 20Ω in the other branch. Determine the current and power dissipated in each branch if the total current drawn by the parallel circuit is 10Ļ-30 ^{0}A



$$Z_1 = 20 + j15$$
$$Z_2 = 30 - j20$$
$$I = 10\angle -30^\circ = 8.66 - j5$$
$$I_1 = I\frac{Z_2}{Z_1 + Z_2} = (8.66 - j5)\times\frac{(30 - j20)}{(20 + j15)+(30 - j20)}$$
$$I_1 = 3.8 - j6.08 = 7.17\angle -60^\circ$$
$$I_2 = I - I_1 = (8.66 - j5) - (3.8 - j6.08)$$
$$I_2 = 4.86 + j1.08 = 4.98\angle -12.5^\circ$$
$$P_1 = I_1^2 R_1 = 7.17^2 \times 20 = 1028.2W$$
$$P_1 = I_2^2 R_2 = 4.98^2 \times 30 = 744W$$

Problem 14

A non inductive resistor of 10Ω is in series with a capacitor of 100µF across a 250V, 50Hz ac supply. Determine the current taken by the capacitor and power factor of the circuit

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 100\times 10^{-6}} = 31.83\Omega$$
$$Z = R - jX_C = 10 - j31.83$$
$$I = \frac{V}{Z} = \frac{250}{10 - j31.83} = 2.24 + j7.14 = 7.49\angle 72.5^\circ$$
$$\phi = 72.5^\circ$$
$$pf = \cos\phi = \cos 72.5^\circ = 0.3$$

Problem 15

An impedance coil in parallel with a 100µF capacitor is connected across a 200V, 50Hz supply. The coil takes a current of 4A and the power loss in the coil is 600W. Calculate (i) the resistance of the coil (ii) the inductance of the coil (iii) the power factor of the entire circuit.

$$Z_{coil} = \frac{V}{I} = \frac{200}{4} = 50\Omega$$

$$P = I^2 R = 600W$$

$$R = \frac{600}{I^2} = \frac{600}{4^2} = 37.5\Omega$$

$$X_L = \sqrt{Z_{coil}{}^2 - R^2} = \sqrt{50^2 - 37.5^2} = 33.07\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{33.07}{2\times 3.14\times 50} = 0.105H$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 100\times 10^{-6}} = 31.83\Omega$$

$$Z_1 = R + jX_L = 37.5 + j33.07$$

$$Z_2 = -jX_C = -j31.83$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(37.5 + j33.07)(-j31.83)}{(37.5 + j33.07) + (-j31.83)}$$

$$Z = 27 - j32.72 = 42.42\angle -50.5^\circ$$

$$\Phi = -50.5^\circ$$

$$pf = \cos\Phi = \cos(-50.5^\circ) = 0.6365$$

Problem 16

A series RLC circuit is connected across a 50Hz supply. R=100Ω, L=159.16mH and C=63.7μF. If the voltage across C is 150Ļ-90^{0}V. Find the supply voltage

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 159.16\times 10^{-3} = 50\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 63.7\times 10^{-6}} = 50\Omega$$

$$V_C = I(-jX_C) = 150\angle -90^\circ = -j150$$

$$I = \frac{-j150}{-jX_C} = \frac{-j150}{-j50} = 3\angle 0^\circ A$$

$$Z = R + j(X_L - X_C) = 100 + j(50 - 50) = 100\Omega$$

$$V = IZ = 3\times 100 = 300V$$

Problem 17

A circuit having a resistance of 20Ω and inductance of 0.07H is connected in parallel with a series combination of 50Ω resistance and 60µF capacitance. Calculate the total current, when the parallel combination is connected across 230V, 50Hz supply.

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 0.07 = 22\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 60\times 10^{-6}} = 53\Omega$$

$$Z_1 = 20 + j22$$

$$Z_2 = 50 - j53$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(20 + j22)(50 - j53)}{(20 + j22) + (50 - j53)} = 25.7 + \mathrm{j}11.9$$

$$I = \frac{V}{Z} = \frac{230}{Z} = 7.4 - \mathrm{j}3.4 = 8.13\angle -24.9^\circ$$